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Art. I.—*Bacchus and Anti-Bacchus.*

(Concluded from the No. for April, p. 306.)

II. In the examination of the essays Bacchus and Anti-Bacchus, begun in our No. for April, the second position proposed to be considered had respect to the strength of the wines in Palestine. "It is impossible," says Mr. Parsons, "to obtain strong alcoholic cider from sweet apples, and for the same reason *it is impossible to obtain strong wines from very sweet grapes*, but *the grapes* of *Palestine*, Asia Minor, Egypt, &c. were *exceedingly sweet.*" Anti-Bacchus, p. 203. And why is it impossible? Let Mr. Parsons answer. "Thus the sweetness of the fruits and of the juices, together with the high temperature of the climate, must have been fatal to the existence of strong alcoholic wines." p. 204.

It is true, indeed, that the expressed juice of the grape may be so rich in saccharine matter, as to interfere with its undergoing a thorough fermentation; and it is also true that, in this case, the wine will not be so strong as when the juice is less sweet. But before we conclude that a strong wine cannot be produced from "grapes exceedingly sweet," let us inquire whether there is no method of diminishing the sweetness of the must, and of so increasing the fermensible to his God and Judge, and to him alone. "To his own master he standeth or falleth." It would occasion us no regret, if every one should come to the conclusion that it is his duty to abstain from all use of intoxicating drinks; unless he should be led to entertain scruples in regard to the lawfulness of using wine at the table of our Lord. Had this subject been left untouched, and had no rude hand been laid on the memorials of our Saviour's death, we should probably have taken no part in the discussions respecting the lawfulness or unlawfulness of using inebriating drink, content to let every one adopt that view of the subject which he deemed most in accordance with the word of God.

The wonderfull success which at this very time attends the temperance enterprise, calls for the most sincere and devout expressions of gratitude to the author of all good: and while we contend for our own liberty and that of others in matters of meats and drinks, we mean not to insist upon the expediency of using that liberty. We feel not the least difficulty in adopting as our own the words of the apostle: "It is good neither to eat flesh nor to drink wine, nor any thing whereby thy brother stumbleth, or is offended, or is made weak." And again, "If meat make my brother to offend, I will eat no meat while the world standeth."

A. B. Dod

Art. II.—*An Elementary Treatise on Analytical Geometry: translated from the French of J. B. Biot, for the use of the Cadets of the Virginia Military Institute, at Lexington, Va.; and adapted to the Present State of Mathematical Instruction in the Colleges of the United States.* By Francis H. Smith, A. M., Principal and Professor of Mathematics of the Virginia Military Institute, late Professor of Mathematics in Hampden Sidney College, and formerly Assistant Professor in the U. S. Military Academy at West Point. New York and London: Wiley and Putnam. 1840. pp. 212.

The science of Analytical Geometry is one of the most brilliant inventions of modern times. Next to the Calculus, it is the most important contribution ever made to our mathematical knowledge. Its power, as an instrument of investigation, is unrivalled. Nor is it less remarkable for the singular beauty with which it classifies, in their proper relations, an endless number of particular results, than for the facility with which it discovers them.

No other branch of human knowledge is so entirely the product of one man's labours. Other sciences have reached their perfection by slow degrees. The surmises of one generation have become the discoveries of the next. Fractional and ill-arranged truths have preceded integral forms and scientific order. The guiding idea, or, as Coleridge would have called it, "the mental initiative," which is necessary to discover the relations subsisting between the truths which make up any science, and arrange them in their proper order, and without which there can be no science, but only an assemblage of isolated results, has been, in most cases, gradually evolved through the successive labours of many men. One approximation after another, each nearer the truth, has prepared the way for the production of the happy idea which is to crystallize an indigested mass of truths into order and beauty. Astronomy was so ripe for the principal of universal gravitation at the time of its discovery, that the bustling Hooke almost stumbled upon it, and filled the ears of the Royal Society with clamours against Newton for having robbed him of his property. And the previons researches of others, especially of Wallis, had approached so near the Calculus that Newton and Leibnitz divide the glory of its invention. The remote parentage of the calculus of the moderns may indeed be distinctly traced to the "method of exhaustions" of Archimedes. But there was no such preparation for the application of algebraic analysis to define the nature and discover the properties of lines, surfaces and solids. This invention is the sole property of Descartes, and it has conferred upon him an immortality which his more laborious speculations in metaphysics have failed to secure. His mathematical researches, of which he thought little, now constitute the basis of his fame.* His *Geometria*, a quarto tract of 106 pages, is one of the few treatises which mark an epoch in the history of science.

* This great man seems to have been singularly unfortunate. In his own day he was harassed by persecutions, under the charge of atheism, though he maintained that the most certain of all our knowledge, next to our own existence, is the being of a God. And but scanty justice has been meted out to him since. Absurdities have been laid to his charge which he never taught, and others have received credit for discoveries of truth to which he is fairly entitled.

Geometry, until this time, had been confined within narrow limits. Previous to the institution of the school of Plato, it had discussed only the properties of rectilineal figures, the circle, the cylinder, the cone and the sphere. The method of investigation was that which is given in the Elements of Euclid, in which nothing is permitted to be done but the drawing of a straight line or a circle, and nothing is assumed as true but a few elementary principles, denominated axioms. The Platonic school contributed to Geometry three other curves, known as the Conic Sections, the properties of which were investigated in a similar manner. In this school originated also the celebrated problems of the duplication of the cube and the trisection of an angle, the first of which was solved mechanically by Plato, and geometrically by his pupil, Menechme, by the intersection of two parabolas.

The conic sections were a most important addition to the stores of Geometry, but the chief glory of the Platonic school is derived from the invention of the Geometrical Analysis. We have the authority of Proclus for ascribing this invention to Plato himself. According to this method, the problem to be solved is assumed as done, or the theorem to be proved as true, and from the relations established by this assumption a train of reasoning is carried on until we come to some conclusion known to be true or false, possible or impossible. A synthetical proof or solution is then found by returning from the elementary truth or construction to the original assumption. The conception upon which this method rests is a refined one, and the method itself more fruitful in the discovery of truth than any other of the inventions of the ancients. In the hands of Apollonius and Archimedes, it led to those beautiful constructions and demonstrations which excited the astonishment of the mathematicians of the 14th and 15th centuries, who were ignorant of the means by which they were accomplished.

His famous "*cogito, ergo sum*," the starting point of his philosophy, has been misconstrued and derided. He has been made to teach a doctrine respecting *innate ideas* which he expressly disclaims, his true opinion on that subject being nothing more than must be held by every one who would escape from the materialism to which Locke's philosophy was carried in the hands of Condillac. And he has been accused of fatalism, though he was the first to teach the paramount authority, in all our reasonings upon the human mind, of the evidence afforded by consciousness, and to apply this principle in proof of the liberty of our actions. But whatever may be thought of the value of the contributions made by him to our knowledge of the mind, he was indisputably the first to cast off the trammels of authority, and set the example of a proper *method* in mental philosophy. He was a great man among the great men of his age.

But the geometrical analysis of the ancients, though the only tentative method which they possessed for the discovery of truth, and the most valuable of all their inventions, is tedious and elaborate in its processes. It contains no general rules or methods of investigation. The discovery of one truth has little or no tendency to lead to the discovery of another. The preliminary constructions and steps of reasoning to be employed, must depend upon the particular circumstances of each question, and much tact is often required to conduct the investigation to a successful issue. A kind of contrivance is necessary in selecting the affections of the quantities upon which to found the analysis, and in making the proper graphical constructions, which, proceeding upon no general methods, demands for its successful practice only that sort of ingenuity which is no essential part of a philosophical mind. Lagrange or Laplace might be at fault in the solution of a mathematical riddle, which would present less difficulty to some contributor of the Diarian Repository, who had spent his life in poring over particular results instead of studying general principles; even as Napoleon, we doubt not, might have been foiled at fence by many a *petit maitre* of Paris.

The only other general method of investigation known to the ancients, was that which has been called the *method of exhaustions*, the invention of Archimedes. The general object of geometrical science being the measure of extension, it was soon found that the same methods which sufficed for determining the ratios of right lines to each other, or of the areas contained by right lines, failed when the question was respecting the length of a curve, the measure of the space bounded by curve lines, or the volume comprised within a curve surface. Right lines and rectilineal figures are compared with each other on the principle of superposition. Two lines are of the same length, when the one being placed upon the other, they would exactly coincide,—two triangles, paralellograms, or other rectilineal figures, are equal, if it be shown that they can be made to occupy the same space. In the last analysis of our reasonings in elementary geometry, it will be found that they rest upon the idea of equality derived from coincidence in space. But this principle of superposition is obviously inapplicable when we come to consider curve lines, cuvilinear areas, and volumes. In a curve, like the circle, which is of uniform curvature throughout, we might take any portion of it as a linear unit, and determine the ratio which it bears to the whole curve, or any assigned portion of it; but we could not thus, by means of the principle of superposition, solve the general problem of assigning the length of the circumference of a circle, or any other curve, in terms of a right line. The same difficulty prevents the comparison of curvilineal with rectilineal spaces. It was to overcome this difficulty that the method of exhaustions was invented by Archimedes. This method essentially consists in inscribing a rectilineal figure within a curve, and circumscribing another around it, and obtaining thus two limits, one greater and the other less than the required perimeter or area. As the number of sides is multiplied, it is evident that the difference between the exterior and the interior figure, and, *a fortiori*, between either of them and the curve, will be continually diminished. In pursuing this method of approximation, it was found, in some cases, that there was a certain assignable limit towards which the perimeter or area of the inscribed figure tended, as the number of its sides was increased, and that the circumscribed figure tended to the same limit. This limit was taken to be the perimeter or area of the intermediate curve. It was thus that Archimedes proved that the area of a circle is equal to the rectangle, under its radius and semi-circumference, by proving that this rectangle was always greater than the inscribed, and less than the circumscribed polygon. Any modern mathematician would accept the demonstration founded upon this principle as sufficient, but the ancients always felt it necessary to strengthen it by means of the "*reductio ad absurdum.*" But the cases are comparatively few in which such a limit can be found. When, for instance, the length of the circumference of the circle is sought, it is impossible to determine any line which shall be constantly greater than the perimeter of the inscribed, and less than that of the circumscribed polygon. The only resource in such cases is to approximate to the value sought, by increasing the number of sides of the interior and exterior polygons, and thus diminishing the difference between them, and of course between either of them and the intermediate curve. It was thus that Archimedes, by inscribing and circumscribing a polygon of 96 sides, discovered the approximate ratio of the circumference of a circle to its diameter, to be as 22 to 7, a result which is too great by the 800th part of the diameter, but of which, nevertheless, this greatest of the ancients was so proud that he directed it to be engraved upon his tomb.

This method of investigation, though subtle and ingenious, laboured under very serious difficulties. Like the Geometrical Analysis it furnishes no general methods, so that the discovery of one truth puts us in no better condition for discovering another. The reasoning, too, is in all cases indirect, and the demonstrations to which it leads are so involved and difficult, that without some more compendious and effective instrument of research, science must ever have remained in its infancy. The ancient geometers succeeded in discovering and demonstrating the chief properties of rectilineal figures, the circle, and the five regular solids. When we add to this an imperfect investigation of the conic sections, the cissoid, the conchoid, the quadratrix of Denostratus, and the spiral of Archimedes, we have the sum of the ancient geometry. But instead of wondering at the fragmentary and imperfect character of abstract science among the ancients, our wonder ought rather to be, that with such feeble instruments they were able to accomplish so much. That their methods were not more general and powerful was a necessary consequence of the early state of science; that with these methods they were able to reach so many valuable results, is in the highest degree creditable to their skill and subtlety.

From the decline of Grecian science until the seventeenth century, a period of nearly two thousand years, geometry made no considerable progress. The Romans were incapable of appreciating what the Greeks had done, much less of adding to it; and the Arabs did nothing more than to translate the works of the Greek geometers. In the same state in which Archimedes and Apollonius had left it, the science came into the hands of Descartes, but it left them completely revolutionized. Before the time of Descartes algebra had been applied to geometry by Bombelli, Tartaglia, and especially by Vieta, in his treatise on angular sections. But they had applied it only to the solution of determinate problems, and derived from it no advantage, except in the greater brevity and power of the language with which it furnished them.* The general method of representing every plane

* The following illustration will put the reader in possession of the difference between a *determinate* and an *indeterminate* problem. Suppose the problem

curve by an equation between two unknown quantities, and deducing all its properties by algebraic operations upon this equation, is unquestionably the sole invention of Descartes. No hint of it is to be found in any previous writer; and they who have adduced the algebraic solutions of geometrical problems given by Vieta and others, in disparagement of the claim of Descartes, have shown thereby that they had not penetrated the real spirit of the Cartesian geometry.

In attempting to explain the fundamental conception of the modern geometry, it will be necessary, in the first instance, to establish the possibility of translating, in all cases, considerations of a geometrical nature, into such as shall be purely analytical. There is no apparent connexion, at first sight, between geometrical forms and analytical equations; and yet a little reflection will show that it is in all cases possible to substitute pure considerations of *quantity* for those of *quality*, and thus bring the whole science of geometry within the range of analysis. All our geometrical ideas may be distributed into the three classes of magnitude, form, and position. No ideas can enter into any geometrical question which are not comprehended in one of these three categories. The first of these presents no difficulty. The ratios of magnitudes to each other are expressed by numbers, and come properly within the scope of algebraic representation and analysis. The second class of geometrical ideas, those which relate to form, may be always reduced to the third, since the form of a body must of necessity depend upon the mutual position of the different points of which it is composed. The form of a triangle is completely determined, if the place of every point on its three sides is known; and so of any other figure. The idea of form, in its widest extent, is evidently comprised in that of position, since every affec-

to be, "upon a given line as a base to construct a triangle of which the other two sides shall be equal to two given lines;" it is evident that the conditions are sufficient to determine the triangle in magnitude and position; and the problem is said to be determinate. The vertex of the triangle would be at the intersection of the two circles described around the extremities of the base as centres, with the given lines respectively as radii. But if the base be given, and the vertical angle; and it be required to find the vertex of the triangle, it is evident that an infinite number of points may be found which would satisfy the conditions. Suppose the vertical angle to be a right angle, then since every angle contained in a semicircle is a right angle, if we describe a semicircle upon the given base, every point in this semicircle will be the vertex of a triangle which will fulfil the conditions of the problem. The problem in this case is indeterminate, and the semicircle upon which the required point is situated is called the *locus* of the point.

tion of form may be made to depend upon an affection of place. The preliminary difficulty then which seems to lie in the way of subjecting geometry to the analytical operations of algebra, is reduced to the simple question of representing, in all cases, considerations of position or place, by those of magnitude or quantity.

In showing how to effect this representation, and thus flashing a sudden light over the whole field of geometry, Descartes did nothing more than to generalize a method which is every day used, even by the most ignorant. Whenever we wish to indicate the situation of an object, the only means which we can employ is to refer it to other objects which are known; and this reference is made by assigning the magnitude of the geometrical elements which connect the unknown with the known. Thus we determine the place of any point on the surface of the earth by its distance from the equator, and from another fixed line chosen as a first meridian. Or if one point be determined, we can assign the place of any other, provided its bearing and distance from the known point be given. These two common methods of defining the position of a point on the surface of the earth are complete illustrations of the two kinds of construction most used in analytical geometry. The methods are obviously susceptible of universal application. Let us call the geometrical elements whatever they may be, which make known the position of a point, the *co-ordinates* of the point, the name imposed upon them by Descartes, and continued by all his successors. The co-ordinates of a point upon a plane are evidently two in number. The position of any point upon a plane is determined if we know its distances from any two fixed lines, not parallel to each other, in the same plane. These distances are the *rectilineal* co-ordinates of the point; and the two fixed lines, which are generally taken perpendicular to each other, are termed the *axes*. We may also fix the position of a point upon a plane, provided we know its distance from a fixed point, and the angle made by the line of direction of this distance with a fixed line. These two elements, the distance of the point, and the angle contained between its line of direction and the fixed line, are the *polar* co-ordinates of the point. An infinite number of other systems, besides those of rectilineal and polar co-ordinates for determining the position of a point, may be imagined, but these are the only two systems that are of extensive use. But whatever may be the system of co-ordinates adopted, it is evident that by means of them we may in all cases, make ideas of position depend upon simple considerations of magnitude, since we may represent always a change of place in a point by variations in the numerical value of its co-ordinates.

Having thus shown that all ideas of position, and, consequently, all our elementary geometrical notions, may be reduced to simple numerical considerations, it will be easy to conceive the fundamental idea of Descartes, relative to the analytical representation of geometrical forms. It is at once evident, from the account which has been given of the manner of representing analytically the position of a point upon a plane, that when a line has been defined by any characteristic property which it possesses, this definition will give rise to a corresponding equation between the variable coordinates of the point which describes the line. If a point be supposed to move irregularly upon a plane, its two coordinates being connected by no relation, will be independent the one of the other. But if the point moves, subjected to such a condition as to make it describe any definable line, it is plain that its two co-ordinates will have, throughout its course, a constant and precise relation to each other. This relation may be expressed by a corresponding equation between the co-ordinates, which will be an exact and rigourous definition of the line, since it will express an algebraic property which belongs exclusively to all the points of this line. The numerical relation which, for every point upon the line, exists between its co-ordinates, may be in some cases difficult to discover; but it is clear, from general considerations, that such a relation must exist, even though we should be unable, in any particular case, to determine its precise nature, and express it by means of an equation. One of these co-ordinates we know must be a *function* of the other, though the form of this function may not be in every case assignable.* These considerations seem sufficient to show,

* One quantity is said to be a function of another when they are so related that the value of the one depends upon the value of the other. Thus the space passed through by a falling body is a function of the time of descent: the length of the circumference of a circle is a function of its radius: and, in general, y is a function of x, if the value of y depends in any manner upon the value of x. There are many cases in which it can be shown that one quantity is a function of another, though we are not able to assign the precise form of the function, and others still in which we can determine the analytical form of the function, but are unable to find its calculable value. The object of every department of natural science is to determine the relations subsisting between the phenomena

in its widest extent, the possibility of defining any curve by means of an equation between the co-ordinates of every point situated upon the curve. And this equation will so exactly and completely represent the curve, that the one can receive no modification, however slight, without producing a corresponding change in the other. Every property of the curve will be implicitly included in its equation, and may be deduced from it by proper analytical operations.

We have, for the sake of simplicity, confined the illustration of the leading principle of the modern geometry to the case of curves, all the points of which lie in the same plane. Since every such curve may be represented by an equation between two co-ordinates, the discussion of their properties is termed geometry of two dimensions. A similar course of reasoning would show that, as the position of a point in space is completely determined when we know its distances from three fixed planes, no two of which are parallel to each other, we may define any curve of double curvature, or any surface, plane or curved, by means of an equation between the three co-ordinates of every point upon the curve or surface. The definition, or the mode of genesis, of the curve or surface will express a property common to every point upon it, and the algebraic expression of this property, in terms of the three co-ordinates, will constitute its equation. We thus have a geometry of three dimensions.

We have attempted thus to state, and to justify, upon general principles, independently of its application to this or that particular case, the conception upon which Descartes founded his geometry. There is not in the whole range of science a conception that has been more fruitful in results. It would be difficult to overrate its importance in a scientific view. Immediately upon its announcement geometry passed beyond the narrow limits which had hitherto circumscribed it, and entered upon a career which can never be exhausted. Nor did geometry alone profit by this fertile discovery. The science of rational mechanics was remodelled by it, physical astronomy derived from it inestimable advantage, and it is at this day lending its aid to almost every department of natural philosophy. It has afforded substantial help to experimental science by giving the means of constructing and

which it considers, or to discover the form of the functions which connect them. The moment this is done, the scicnee passes into the hands of analysis, and takes a rational form.

expressing those partial hypotheses, which, prior to the discovery of a complete theory, are necessary to classify the facts that are already known, and guide to the investigation of new ones.

In comparing together the ancient and the modern geometry, it is impossible not to be struck, in the first instance, with the great advantage possessed by the latter in its language. This advantage is so striking that some writers have been deceived into making it the essential distinction between the two methods. All mathematical language consists of two parts; the one expressing the objects themselves about which we reason, the other expressing the manner in which these objects are combined or related, or the operations to which they are subjected. In the ancient geometry magnitudes are represented by *real* symbols, a line by a line, an angle by an angle, a triangle by a triangle, &c.; and the relations of these magnitudes to each other, and the operations to be performed upon them, are described in words. In the modern geometry, on the contrary, the magnitudes about which we reason, the relations which they bear, and the operations to which they are subjected, are all denoted by *conventional* symbols. These symbols are simple, brief, and comprehensive. Instead of a diagram, sometimes exceedingly complicated, accompanied by an enunciation of the truth to be proved, often awkwardly expressed because of the limitations by which it must be guarded, and a demonstration which brings the matter slowly and in successive portions before the mind, we have in the symbols and operations of algebra, as applied to geometry, so much meaning concentrated into a narrow space, expressed with such distinctness and force, and brought with such entireness to the notice of the mind before the impression made by one part has been weakened, that the reasoning powers cannot but be greatly aided, and guarded against error. These symbols afford us also the means of simplifying all the operations to be performed. By means of them we are enabled to reduce all possible relations between the objects of our reasoning to the simplest of those relations, that of equality; and a still more important advantage is gained in the substitution which we are able to make of the arithmetical operations of multiplication and division, instead of the geometrical method of the composition and division of ratios.

But immense as is the superiority conferred upon the modern geometry by the comprehensiveness and power of its language, it is not in this that its essential spirit resides. Without the aid of this language it never could have reached its present state of perfection; but we are not entitled therefore to infer that its peculiar character is derived from the symbols it employs. The use of these symbols, or of others possessing a like simplicity and concentration of meaning, was essential to the development of the science as we now have it, but its logical character is independent of its language. This language may be, and often is, applied to the solution of determinate problems in geometry, which possess, nevertheless, the character of the ancient geometry; and it is possible, on the other hand, to apply, in some cases, the substance of the modern method without the use of its peculiar notation. A little reflection upon the spirit of the two methods will be sufficient to show, that any independent investigation of a particular truth, whether conducted by means of graphical constructions representing by real symbols the quantities about which we reason, or by algebraic characters and processes,—that is, that any special result which is obtained in any other way than by the application of some more general truth to the particular case, belongs essentially to the ancient method in geometry. The ancient geometry is, in other words, an assemblage of particular results; the modern geometry is a collection of general truths, each comprising under it an endless number of particulars.

We have spoken of geometry as the science which has for its object the *measure* of extension. This definition, though it may seem at first sight, by its precision to limit the scope of geometry, does in reality require, for the absolute perfection of this science, that it should discuss all imaginable forms of lines, surfaces and volumes, and discover all the properties which belong to each form.* This statement immediately suggests two essentially distinct modes of investigation; the one by taking up, one by one, these geometrical forms, and determining separately all the properties of each; the other, by grouping together the discussion of analogous properties, no matter how different in other respects may be the bodies† to which they belong. In other words, our geometrical researches may be conducted, and the results of

* For a lucid exposition of this and some other points briefly discussed in this article, the reader is referred to M. Comte's *Cours de Philosophie Positive*, *Lecon* 10e.

† We use the term *body*, for convenience sake, to designate the objects of geometrical study, lines, surfaces and volumes.

them arranged in relation to the different bodies which are the object of study, or in relation to the properties which these bodies present. The first of these was the method pursued by the ancients. They studied, one by one, the properties of the straight line, the circle, the ellipse, the hyperbola, &c., separating the different questions pertaining to each from those which related to other curves or surfaces, no matter how strong the analogies might be between them. This method of investigation, though simple and natural, is obviously characteristic of the infancy of science. The complete mastery of the properties of one curve affords no aid for discovering those of another, beyond the skill and tact which the previous study has imparted. No matter how similar may be the questions discussed respecting different curves, the complete solution of them in relation to one leaves us to commence the investigation anew for every other. However similar a problem may be to one already solved for some other curve, we can never be certain beforehand that we shall have sufficient address to solve it under its modified form. Though we may, for example, have learned how to draw a tangent to an ellipse or hyperbola, this gives us no aid in determining the tangent to any other curve. Geometry, thus studied, is, as we have already called it, evidently nothing more than a collection of particular results, destitute of those general classifying truths which are necessary to constitute a science.

The modern geometry, on the other hand, instead of investigating *seriatim* the properties of each geometrical form, groups together all affections of a like kind and discusses them without regard to the particular bodies to which they belong. It passes over, for instance, the particular problem of finding the area of the circle, and solves the general problem of finding the area bounded by any curve line whatever. Instead of investigating the asymptote to the hyperbola, and then remaining in no better condition than before for discovering whether any new curve has asymptotes or not, it puts us in possession at once of a general method for determining the asymptotic lines, straight or curved, which belong to any curve whatever. The modern geometry treats thus, in a manner perfectly general, every question relative to the same geometrical property or affection, without regard to the particular body to which it may belong. The application of the general theorems thus constructed, to the particular circumstances of this or that curve or surface, is a work of subordinate importance, to be executed accordingly to certain rules that are invariable in their mode of application and infallible in their promise of success.

Let any new curve be proposed to one who is destitute of the resources of the modern geometry, and he must commence first by surmising, and that chiefly through the suggestive power of graphical constructions, what its properties are, and then endeavor to prove by methods altogether peculiar to the curve in hand, that it possesses the properties the existence of which he has divined, with no certainty derived from his previous knowledge that he will be able to succeed in this particular case. Foiled amid its intricate specialities he may be reduced, as was the great Galileo, to the mortifying necessity of calling in the mechanical aid of the scales to supply the defect of his mathematical resources.* Let the same curve be proposed to one who has the modern geometry at command, and he will immediately determine its tangent, its singular points, its asymptotes, its radius of curvature, its involute and evolute, its caustics, its maximum and minimum ordinates, its length, its area, the content of the solid generated by its revolution, in short all its important properties.

The brief exposition which we have given of the different methods pursued by the ancient and the modern geometry, is enough to show on which side the scientific superiority lies. In the ancient geometry special results are obtained separately, and without any knowledge of their mutual relations though they may be, in truth, only particular modifications of some general truth which embraces them and innumerable like phenomena. The modern geometry investigates this general truth, and then applies it, in the way of deduction, to all particular cases. Had we gone on for ages in the steps of the ancients, we could have done nothing more than add to the *indigesta moles* of particular truths; and no matter how great our success there would still always remain an infinite variety of geometrical forms unstudied and unknown. On the other hand, for every question resolved by the modern geometry, the number of geometrical problems to be solved

* The only stain upon the scientific reputation of this great man is his seeking to determine the area of the cycloid in terms of its generating circle, by cutting the cycloid and the circle out of a lamina of uniform thickness and weighing them. It is a striking illustration of the power of the modern analysis that any tyro can now solve problems that eluded the forces of such men as Galileo, Fermat, Roberval, and Pascal.

is diminished, for all possible bodies. The one is a science, with its general theorems lying ready for all possible cases; the other is made up of independent researches, which, when they have gained their particular end, shed no light beyond it.

It is not our purpose to enter fully into the exposition of the peculiar logic of the modern analysis, or to contrast in detail its merits with those of the ancient geometry. Many interesting points of view could be obtained by pursuing this comparison to a greater length; but we have gained the end which we at present have in view if we have given an exposition of the subject sufficiently plain and extended to enable the reader to pronounce upon the *scientific* claims of the two methods. We entertain no doubt what will be the judgment rendered.

The superiority of the analytical methods of the moderns is so evident and vast, that there has been no attempt, since the publication of the "Geometry of Curve Lines," by Professor Leslie, to revive the ancient method. This attempt was a signal failure. Mr. Leslie avows himself the champion of a juster taste in the cultivation of mathematical sciences, but unfortunatcly for his success, no sooner does he enter upon any question which lies beyond the mere elements of geometry than he betrays most painfully the poverty of his resources. We have but to open his book and read of "a tangent and a point merging the same contact," of points "absorbing one another," of "tangents melting into the curve," of "curves migrating into one another," &c., to make us sympathize with the humiliation which he must have felt in invoking the aid of poetry to establish the theorems of geometry. We know of no similar attempt made by any *scholar* since. It is now universally conceded that without the aid of the modern analysis, the science of geometry cannot be established upon a rational basis. And without the help of geometry, thus established and ordered, all the real sciences, excepting only those included in the department of natural history, must be deprived of their full developement and perfection. The new geometry has its ample vindication in the "Mecanique Analytique" of Lagrange, and the "Mecanique Celeste" of Laplace.

In our own country, prior to the publication of the work named at the head of this article, we had but two treatises on the subject of Analytical Geometry; the one a republication of the elementary treatise of Mr. J. R. Young, which is chiefly made up from the "Application de l'Algebre a la

Geometric" of Bourbon; the other, a more recent publication from the pen of Prof. Davies. We do not, for reasons that will be obvious enough, include among treatises upon Analytical Geometry, the Cambridge translation of the imperfect and antiquated work of Bèzout. We are glad that Prof. Smith has added his contribution to our scanty stock, by giving us a translation of the masterly work of Biot, one of the most perfect scientific gems to be found in any language. The original needs not our commendation, and of the translation it is enough to say that it is faithfully executed.*

We regard the multiplication of text books, on this subject, as affording cheering evidence that juster ideas are beginning to prevail in our country respecting the proper scope of mathematical education. And yet there are colleges in our land that comprise, in their course of study, nothing of the geometry of curves beyond what is contained in Simpson's or Bridge's Conic Sections, that leave the study of the Calculus optional with the student, and that are compelled, therefore, to teach, under the name of Natural Philosophy, a system that, at the present day, is scarcely level with the demands of a young ladies' boarding school. The graduates of these institutions may be able to classify plants, insects and stones; they may fancy themselves qualified to decide upon the comparative merits of rival systems of world-building in geology; but they cannot read, understandingly, the first ten pages of any reputable treatise on mechanics from the French or English press. We have grieved long over this state of things, and we hail with pleasure every symptom of a change for the better in public sentiment. If our ancient and venerable institutions of learning will not elevate their course of study into some approximation to the existing state of mathematical science, the day, we hope, is not far distant when the public will discern that they are standing in the way of a thorough education, and visit them accordingly.

* We regret to see so many typographical errors in the work, and some of them of a character fitted to perplex the student. On page 88 there is an omission of the transformation of the equation of the Ellipse, to remove the origin from the vertex of the axis to the centre of the curve, which confuses all the subsequent investigation.